Limits to Infinity and Intro to Derivatives

Math 130 - Essentials of Calculus

17 February 2021

Examples

Example

Compute the limits

1. $$\lim_{x\to\infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4}$$

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4. $$\lim_{x\to\infty} \frac{x+x^3+x^5}{1-x^2+x^4}$$

Asymptote Example

Example

Find all vertical and horizontal asymptotes of the curve

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2}$$

Instantaneous Rate of Change

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$$\lim_{\Delta t \to 0} \frac{\Delta h}{\Delta t} = \lim_{t \to 1} \frac{h(t) - h(1)}{t - 1}.$$

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Definition (Instantaneous Rate of Change)

The instantaneous rate of change of a function f at the input value x_1 is

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

provided the limit exists.

Instantaneous Rate of Change

An alternative, but equivalent definition for the instantaneous rate of change is

Definition (Instantaneous Rate of Change)

The instantaneous rate of change of a function f at the input value a is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

Think of $x_1 = a$, $x_2 = a + h$, then $\Delta x = h$.

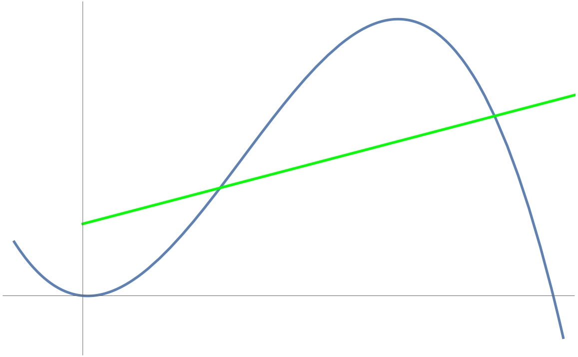
Instantaneous Rate of Change

Example

A rock is dropped from a bridge over a river. The distance, in meters, between the rock and the fiver t seconds after the rock is dropped is given by $s(t) = 48 - 4.9t^2$. Compute the speed of the rock after 2 seconds.

Slope of the Tangent Line

Recall that the average rate of change was the slope of a *secant line*. As we shrink the interval that the average rate of change is taken over, the slope of the secant lines approaches the slope of the tangent line.



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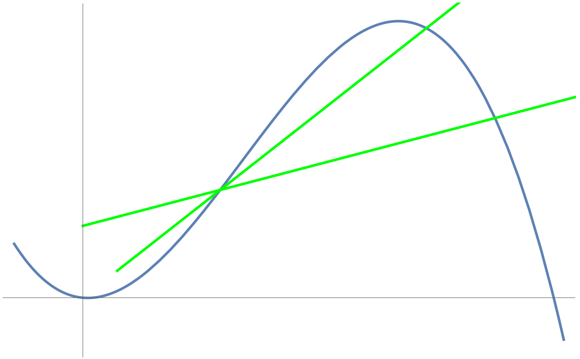

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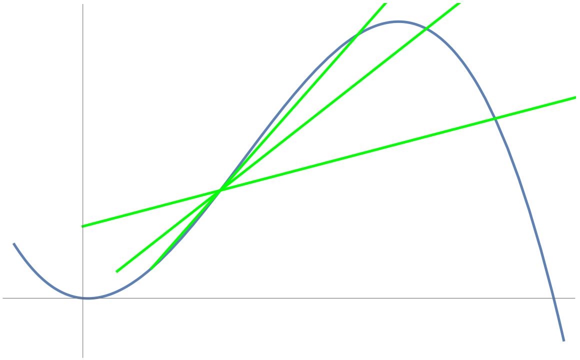

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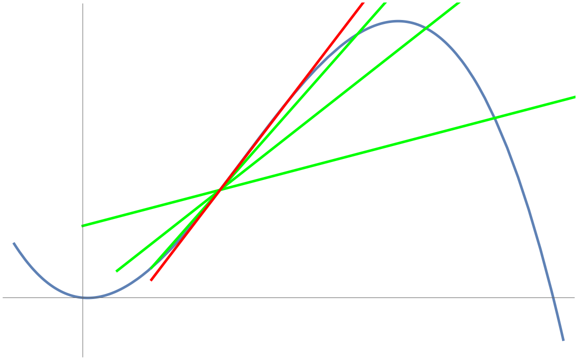

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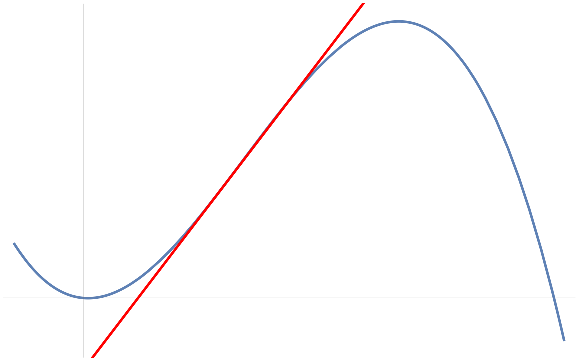

Slope of the Tangent Line

Definition (Slope of Tangent Line)

The tangent line to the curve $y = f(x)$ at the point $(x_1, f(x_1))$ is the line though this point with slope

$$m = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

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$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

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Finding the Tangent Line

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Find the equation of the tangent line to the given function at the given point:

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1. $y = 2x^2 + 1$ *at* $(3, 19)$
2. $f(x) = 3x - x^2$ *at* $(1, 2)$